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MATHEMATICAL, PHYSICAL & ENGINEERING SCIENCES

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Phil. Trans. R. Soc. Lond. A 1980 **294**, 299-305
doi: 10.1098/rsta.1980.0036

Phil. Trans. R. Soc. Lond. A **294**, 299–305 (1980)
Printed in Great Britain

Geoid float techniques in satellite geodesy

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This approach to reconciling satellite-derived positions of ground stations with terrestrial geodetic data avoids uncritical acceptance of existing astro-geodetic or other geoidal separation information but rather allows such a geoid to 'float' up and down freely to find its own best position.

This preserves geoidal shape but changes its absolute position so as best to harmonize the satellite–terrestrial geodetic positions with some adopted datum shift over the area.

The underlying philosophy is discussed in relation to existing published results; the effect of 'wrong' adopted datum shift (and the 'geoid match' technique to remedy it) is discussed.

1. The problem

The problem of marrying ground station positions derived by Doppler satellite geodesy into terrestrial geodetic networks is made unnecessarily untidy by the undiscriminating use of an astro-geodetic geoid for converting the heights above mean sea level of classical ground survey into heights above the spheroid of reference. In spite of Oscar Wilde, who wrote, 'Truth is never pure, and rarely simple... In married life three is company and two is none...' (*The Importance of Being Earnest*, 1894), this triangular interaction of satellite data, ground survey data and geoidal separations can and should be separated into its component parts to bring to light the pure and simple relations between them. This is because there are objections to using (as if it were absolute in datum and error-free in accuracy) an astro-geodetic geoid (such as Bomford (1971) (Moscow) or Levallois (1975) (Grenoble)) to convert the ground survey positions into truly three-dimensional Cartesian positions for direct comparison with the Doppler fixes. Such a geoid often has a fairly arbitrary datum value as its starting point (how good now is the astro-geodetic connection to Potsdam in East Germany?) and must build up some error as it goes along, since it is computed by deriving, line by line, the change in geoidal separation between the end points of each line of the network from the relative slope of geoid and spheroid between them.

Some device is therefore required which shall use the internal logic of the data itself to dictate the form of the solution, very much as was described by Georg Hegel: 'Reason is as cunning as she is powerful, [makes] objects... react [with each] other in accordance with their own nature [so as] to guide things toward her own ends' (*Encyclopaedia*, 1840), so that the errors and inconsistencies, if any, between the Doppler and the geoid data will 'separate themselves off', as it were, from those between the Doppler and ground survey data, and both be displayed separately.

2. The Doppler–geoid interface

The device adopted is as trivial as it seems effective, and has been called 'geoid float'. It is assumed that the data shown in table 1 are available for a number of points, and the 'published' datum shift is then applied to the Doppler positions to convert them into the local ground survey datum. The resulting latitudes and longitudes are discarded (as of no interest) but the heights above spheroid are converted to geoidal separations and compared with those read from the geoid.

Table 1. Data for geoid float

(1) Doppler: ϕ, λ, h (e.g. on WGS 72)
(2) ground survey: ϕ, λ (e.g. on ED 50)
(3) height above mean sea level
(4) geoid (e.g. that of Levallois (1975))
(5) datum shift (e.g. 84, 103, 127)

If the discrepancies are large and random, with no systematic pattern between those at adjacent points, then it can only be said that there are serious deficiencies in either the Doppler or the mean sea level heights (or both); short of obtaining additional data, it is essentially a matter of judgement which to reject, although there are some indications (see later) to guide the choice.

If, on the other hand, the discrepancies are systematic over the are. , it is an easy matter to allow the geoid to 'float' freely up and down, and also to suffer small systematic distortions of shape so as to eliminate them completely. This is conveniently done by constructing 'correction contours' to overlay the original geoid, with the added advantage that these apply equally to any other geoid (on a different spheroid and/or datum) that is related by datum shift to the original geoid.

To adjust the geoid to fit the Doppler is not to say that the geoid is necessarily wrong and the Doppler right: it is a convenient way to achieve harmony, and 'rightness' in this context is perhaps partly a matter of semantics, although (see later) this is not wholly true.

The main information content of the geoid is its shape (as pointed out earlier, its absolute positioning is somewhat arbitary), and to allow it to float freely up and down to fit the relatively sparse number of Doppler stations will do little or no violence to the local shape of the surface.

The choice of 'published' datum shift for use in the geoid float is surprisingly uncritical: (84, 103, 127) given by Seppelin for converting WGS 72 to European Datum is perfectly adequate in spite of the scanty evidence on which it was originally based.

If, however, a chosen shift proved so far from 'ground truth' as to cause noticeable distortion at the data interface, because a relatively large portion of the Earth's surface was involved, then this very fact would enable the geoid match technique (to be described later) to 'bite' and so derive a better shift.

The data themselves are seen to be providing their own remedy in the Hegelian interplay of thesis, antithesis and synthesis, on the principle described by Hegel's somewhat unlikely disciple (in another context) as: 'From each according to his abilities, to each according to his needs' (K. Marx, *Criticism of the Gotha Programme*).

Discussion has so far been confined to the overall pattern of data behaviour but the method also provides a powerful tool for investigating local anomalies (or 'data snooping' as Professor Baarda would term it). The geoid correction contours derived from Doppler fixes in and around the United Kingdom (see figure 1) were fairly smooth and regular except where a tight little knot of contours in the south became known as the 'grumbling appendix', as it clearly needed surgical attention. This lucky accident that the anomalous station lay within a circle of others resulted in clear evidence that it was out of sympathy by about 2 m either in the Doppler derived height or in the height above mean sea level. The Doppler–ground survey interface

PHILOSOPHICAL TRANSACTIONS OF THE ROYAL SOCIETY A MATHEMATICAL, PHYSICAL & ENGINEERING SCIENCES

(see §3) threw suspicion on the Doppler: the observation was repeated and the anomaly removed.

The corrections to Bomford's 1971 Moscow geoid derived from Doppler fixes in and around Europe (see figure 2) amounted to no less than −30.4 m in the east (Mashad in Iran). It was satisfying to learn that J. J. Levallois (personal communication) had data to extend his 1975 geoid as far as Mashad and that he could confirm the amended value to within a few metres.

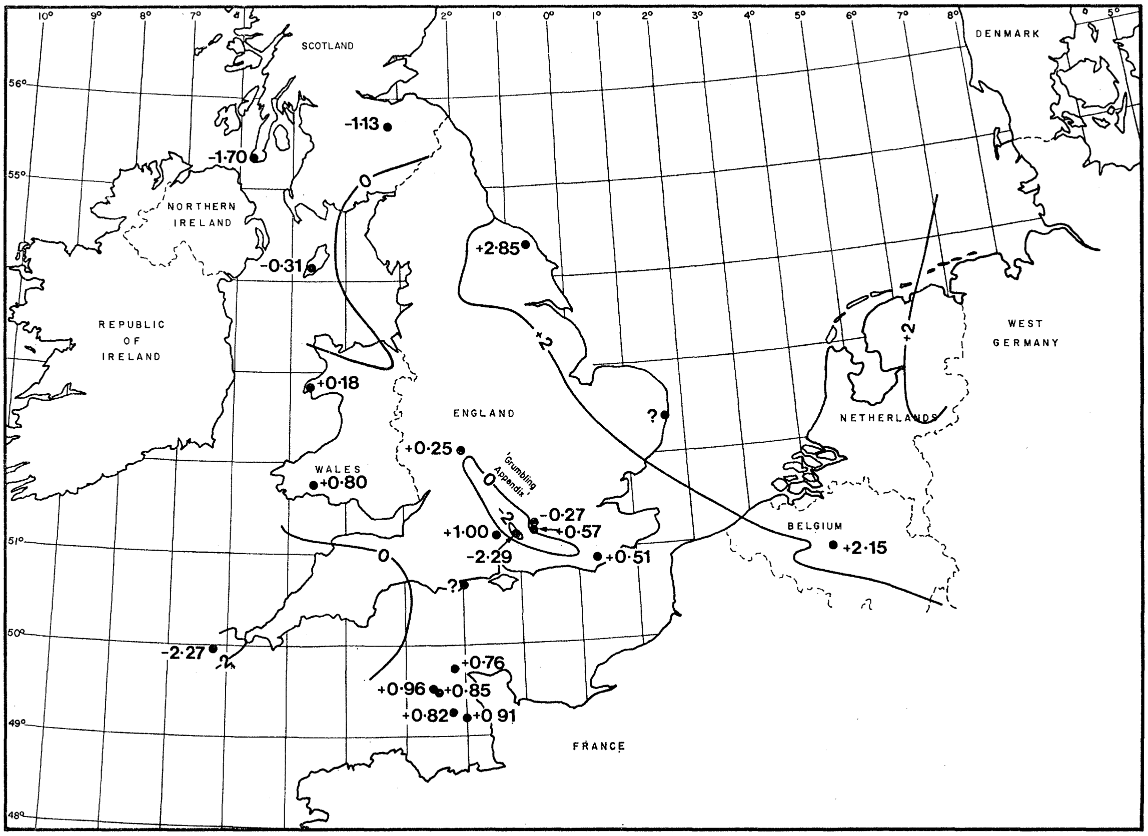

FIGURE 1. Geoid float correction contours for Levallois (1975) geoid, U.K.

In Cyprus (see figure 3) the range of corrections to Bomford's geoid (8.0 m) is reduced to 2.4 m for that of Levallois's, showing that the local shape of the geoid fits the Doppler results very much better when the former incorporates Levallois's additional data.

It is fair to note that the error correction contour interval is about equally spaced in the two cases – the contours being 'rotated', as it were, to produce the greater range – but it is suggested that the improved fit is still genuine.

In East Africa, where two estimates (by Guy Bomford and Irene Fischer) of the geoidal profile along the arc of the 30th meridian were all that was available, geoid float was used (see figure 4) to produce a geoid map covering Kenya, Ethiopia and the Eastern Sudan. After noting the overall consistency of the 5 m internal contours, obtained by making all possible interpolations between the individual geoidal separations derived from the Doppler, one's attention is drawn to the three small 'hiccups' of the order of 3 m or so in the contours. Two of these (at

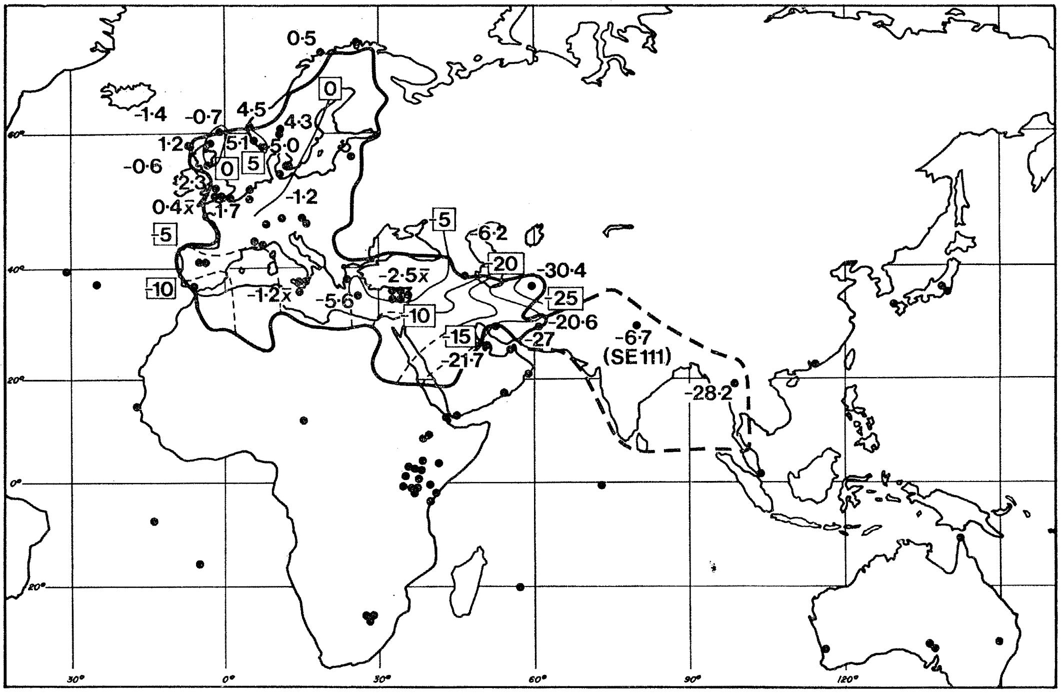

FIGURE 2. Suggested adjustment contours for Bomford's 1971 Moscow geoid. ——, Limits of Bomford's 1971 geoid; – – –, limits of Bomford's related Indian geoid; — - -, adjustment contours.

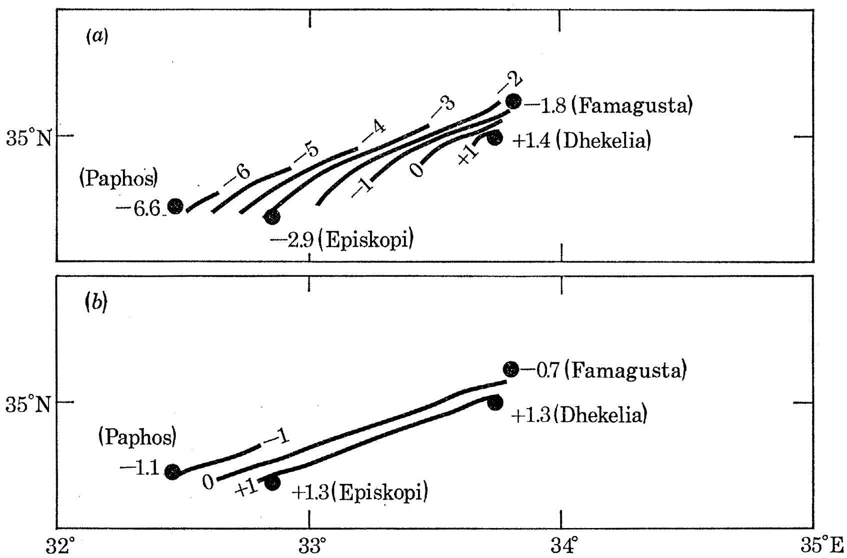

FIGURE 3. Geoid float correction contours in Cyprus:
(*a*) Bomford 1971 geoid; (*b*) Levallois 1975 geoid.

Addis Ababa and south of Lake Rudolf), if not the third (on the Red Sea coast), lie directly on the line of the East African Rift Valley. It is interesting to note that, as confirmed by gravity observations made in Kenya, the deviation of the vertical is in towards the rift rather than, as one would at first sight suppose, outwards from it.

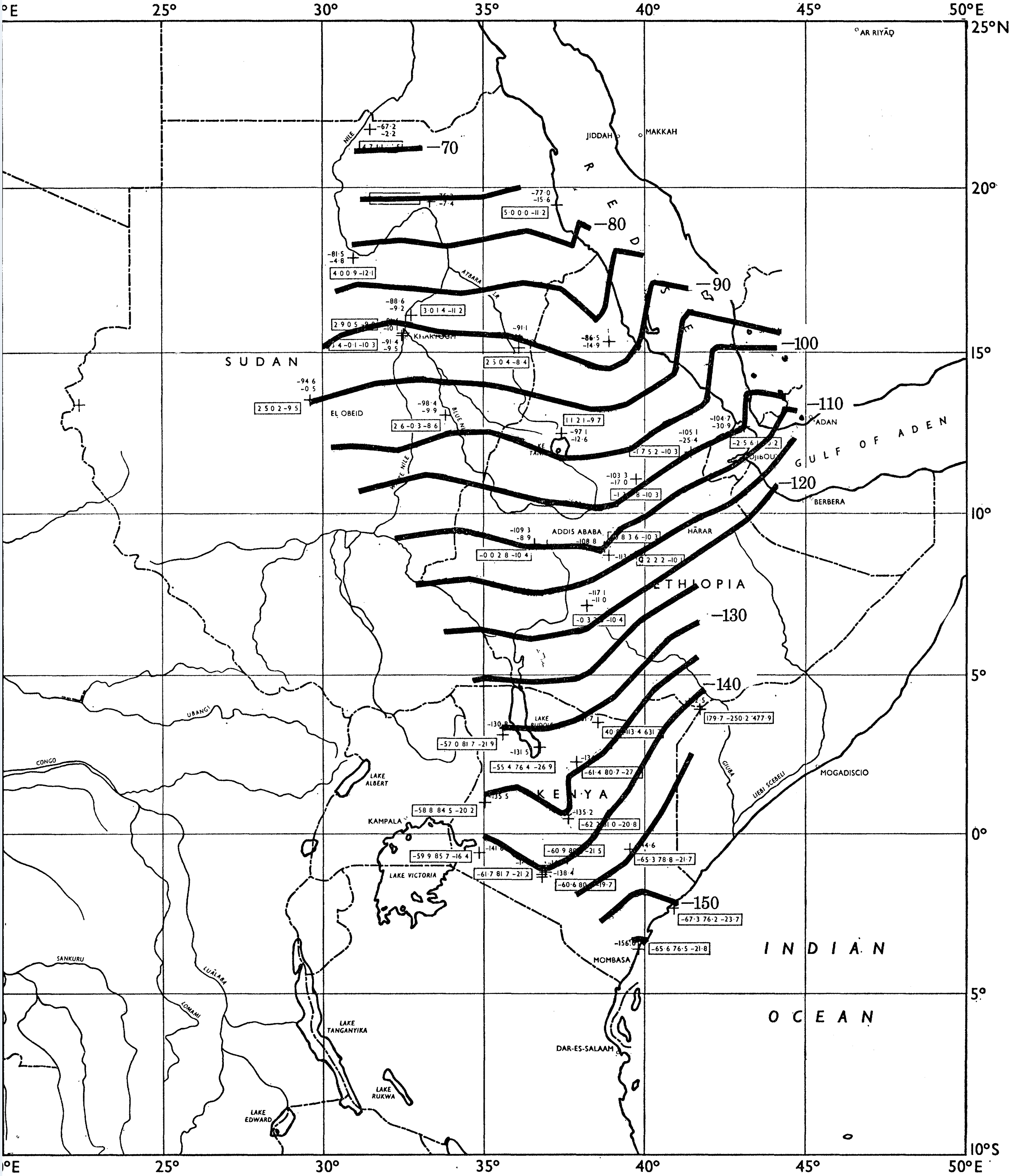

FIGURE 4. Doppler results in Africa: European Datum geoidal separations.

PHILOSOPHICAL TRANSACTIONS OF THE ROYAL SOCIETY A MATHEMATICAL, PHYSICAL & ENGINEERING SCIENCES

3. The Doppler–ground survey interface

In broad terms, this interface represents the horizontal discrepancy after geoid float has ironed out the vertical component. It was after geoid float that it was possible to see a residual scale error of about 3 parts/10^6 in the 1970 scientific adjustment of the Ordnance Survey triangulation from the highly latitude-dependent vector discrepancy of the Doppler–ground survey misfit (table 2). Preliminary results of the U.K. contribution to the second (1977) European Doppler campaign were presented in Luxemburg by Ashkenazi. Geoid float changes these as shown in table 3, and the systematic pattern that he noticed becomes even more apparent.

Table 2. Doppler–ground survey vector discrepancies

latitude, N	discrepancy before scaling/m	discrepancy after scaling/m
60° 26′	3.62	1.32
58° 33′	3.00	2.16
58° 11′	2.23	1.21
55° 44′	2.06	0.93
52° 09′	1.10	0.50
51° 11′	1.00	1.18
50° 52′	(datum)	(datum)

Table 3. Broadcast–precise ephemeris datum shifts

	shifts before geoid float/m			shifts after geoid float/m		
	X	*Y*	*Z*	*X*	*Y*	*Z*
England						
SW	−4.0	5.1	−10.2	2.8	4.6	−2.0
SE	−3.4	4.6	−10.3	2.9	4.6	−2.4
Notts.	−1.2	3.4	−7.3	2.8	3.3	−2.0
NE	−1.0	4.1	−8.8	3.5	3.9	−2.4
Scotland	−0.9	4.9	−9.5	3.7	4.7	−2.3

To consider both interfaces together when investigating individual station irregularities may give some indication of the cause of the trouble. If both interfaces show error, this is likely to lie in the Doppler; if only the geoid–Doppler interface, then the error may be in the geoid; if only the Doppler–ground interface, then the ground survey is a slightly more likely candidate.

4. Geoid match

The effect of a change (u, v, w) in the adopted 'published' datum shift can be expressed (geoid match observation equation) as

$$u \cos \phi \cos \lambda + v \cos \phi \sin \lambda + w \sin \phi = \mathrm{d}h,$$

where $\mathrm{d}h$ is geoid difference at (ϕ, λ), and $\{u, v, w\}$ is datum shift. The equation is solved by least squares for u, v, w.

If an unwise original choice of shift has caused distortion at the geoid–Doppler interface (because the area of the network of points on the Earth's surface is relatively large), the coefficients of this equation will vary significantly from point to point. These equations will then serve as observation equations in a non-singular least squares solution for an improved datum shift. If no such solution is possible, then none will be required. Once more the internal logic of the data is dictating the method to be followed.

5. Literary epilogue

The following extract from Keats's last book of poems (which was in Shelley's pocket when he set sail from the Italian shore):

Philosophy will clip an angel's wings,
Conquer all mysteries by rule and line,
Empty the haunted air and gnomèd mine–
Unweave a rainbow...

Lamia (1821)

must be appropriate for this time and place.

Is not the angel in the haunted air an Earth satellite? The unweaved rainbow is a Doppler pass and what are the inhabitants of the gnomèd mine but R.I.C.S. land surveyors?

References (Weightman)

Bomford, G. 1971 IUGG General Assembly, Moscow.
Levallois, J. J. 1975 IUGG General Assembly, Grenoble.